Large Scale Systems

Chapter 5

Large Scale Systems

Order reduction

version 1.0

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Introduction

Objectives

- Understand the interest of model order reduction
- Apply model order reduction based on Balanced Realization

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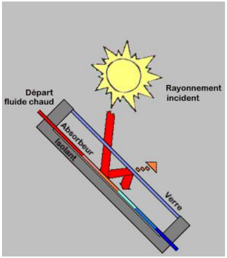
- Understand the interest of model order reduction
- Apply model order reduction based on Balanced Realization

Work

- Consider an example
- Deliver a report

Problem specification

Solar heating of water



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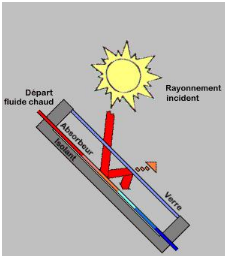

Modelling Asumptions

- The solar panel is considered as a pipe of length L,
- The solar irradiation is constant on the pipe,
- The water flow is constant and fixed.

Modelling

Cell equations

$$\frac{dTw_i}{dt} = \frac{1}{m_{wi} \cdot Cp_w} (h_{CW} \cdot A_{CW} \cdot (Tc_i - Twi) + q \cdot Cp_w (Tw_{i-1} - Twi)) \quad (1)$$

$$\frac{dTc_i}{dt} = \frac{1}{m_{ci} \cdot Cp_c} (h_{CW} \cdot A_{CW} \cdot (Tw_i - Tci) + A_{cr} \cdot F_{abs} \cdot Rad + h_{CW} \cdot A_{CA} \cdot (Ta - Tc_i)) \quad (2)$$

where

- w stands for water, c for copper and a for air, i is the number of the considered cell and $i - 1$ the previous one,
- Cp_x is the specific heat, mx_i the mass, q the water mass flow rate,F_{abs} is the absorbing factor of the pipe and Rad is the solar irradation
- Tx_i stands for the temperature, A_{xy} for the exchange area and h_{xy} the exchange coefficient.

Modelling

Considering N cells

$$
\begin{aligned}
\frac{X}{dt} &= A \cdot X + B \cdot u \\
T_{out} &= C \cdot X + D \cdot u
\end{aligned}
\tag{3}
$$

where

- $X = (Tw_N, Tc_N, Tw_{N-1}, \ldots Tc_1)'$
- $u = (Rad, Ta, T_{in})'$
- T_{out} the ouput and T_{in} the input temperature of water.

Modelling

Considering N cells

$$\begin{aligned} \frac{X}{dt} &= A \cdot X + B \cdot u \\ T_{out} &= C \cdot X + D \cdot u \end{aligned} \quad (3)$$

where

- $X = (Tw_N, Tc_N, Tw_{N-1}, \ldots Tc_1)'$
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PanneauDyn Matlab function

- input: N the number of cells
- output: state space system associated to (3)

1 - Spatial discretization

Similarity of step response of the approximations will be used to assess them

1. compute the spatial discretization of the system considering various numbers of cells [1]. The numbers should at least include: 1, 2, 5, 10, 100, 200, 300.
2. compare the systems and propose a good value for the number of cells to be considered in the spatial discretization.

[1] From now on, the system obtained with N cells will be denoted SN

2 - Hankel Singular Values

1. give the definition of controlability and observability gramians of a system and a brief interpretation of these gramians. Compute the gramians for S2.
2. give the definition and interpretation of the *balanced realization* of a system. Compute the balanced realization [2] of S2 and check its gramians.
3. What are the Hankel Singular Values of a system and give the Hankel Singular Values of S2

Tips

Matlab functions: gram, balreal, hsvd

[2] The balanced realization of SN will be denoted SBN

3 - Balanced realization order reduction

1. Explain the principle of model order reduction based on balanced realisation
2. Consider the Hankel Singular Values of S200 and propose an order (n) that is relevant to reduce the order of the system.
3. Compute the Balanced Realization Reduduction of S200 at order n (SR200.n).
4. Compare the systems S200, Sn and SR200.n.
5. Compare the systems S200 and SR200.(n/2).

Tips

Matlab functions : balred

4- Conclusion

Conclude on this study and propose other methods to reduce the order of models.